Implicit differentiation, equation of the tangent line

Using implicit differentiation to find the equation of the tangent line is only slightly different than finding the equation of the tangent line using regular differentiation. Remember that we follow these steps to find the equation of the tangent line using normal differentiation:

1. Take the derivative of the given function.
2. Evaluate the derivative at the given point to find the slope of the tangent line.
3. Plug the slope of the tangent line and the given point into the point-slope formula for the equation of a line, $(y - y_1) = m(x - x_1)$, then simplify.

This result is the equation of the tangent line to the given function at the given point. When we have a function that isn't defined explicitly for y, and finding the derivative requires implicit differentiation, we follow the same steps we just outlined, except that we use implicit differentiation instead of regular differentiation to take the derivative in Step 1.

Example

Find the equation of the tangent line at the point $(1,2)$.

$$3y^2 - 2x^5 = 10$$

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We'll use implicit differentiation, since solving our equation for y is a little tedious and gives us an ugly value. Remember that whenever we take the derivative of a term involving y, we have to multiply our result by the derivative of y, which we can write as dy/dx or as y'.

$$6y \cdot y' - 10x^4 = 0$$

Now we'll simplify and solve for y'. (Note: If we can't easily solve for y', it's okay. We can just plug in the given point and *then* solve for y'.)

$$6yy' = 10x^4$$

$$y' = \frac{10x^4}{6y}$$

$$y' = \frac{5x^4}{3y}$$

Now we'll plug in the given point, $(1,2)$, to find the slope of the tangent line at that point.

$$y'(1,2) = \frac{5(1)^4}{3(2)}$$

$$y'(1,2) = \frac{5}{6}$$

$$m = \frac{5}{6}$$

Now we can plug the point $(1,2)$ and the slope $m = 5/6$ into the point-slope formula for the equation of the line,

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$$y - y_1 = m(x - x_1)$$

The equation of the tangent line to $3y^2 - 2x^5 = 10$ at $(1,2)$ is

$$y - 2 = \frac{5}{6}(x - 1)$$

$$y - 2 = \frac{5}{6}x - \frac{5}{6}$$

$$y = \frac{5}{6}x - \frac{5}{6} + \frac{12}{6}$$

$$y = \frac{5}{6}x + \frac{7}{6}$$

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